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TOPIC 3

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3.3.2 Time-dependent methods and transient analysis

Reactor transient analysis: from point kinetics to the quasi-static method

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Introduction

Accurate description of non-equilibrium reactor behaviour has always been - and still is - a major concern for the assessment of safety. In early times, time-dependent neutron transport solutions were obtained using *ad-hoc* models like the point-kinetics (PK) equations which, despite the fact that they were approximate, had strong physical relevance [1]. There were essentially two motivations for this: limited computing facilities and a lack of adequate numerical schemes to solve very large systems of *stiff* ordinary differential equations. The PK model was based on the - notoriously wrong - assumption of separable time- and space dependence of the neutron flux solution to the basic equations. This model was quite relevant for tightly coupled systems – *i.e.* fast reactors - considered in the early fifties. In particular, it contained most of the physical features of these systems within a reduced set of parameters: reactivity, effective delayed neutron fraction, prompt neutron lifetime, etc… However, the emergence of less tightly coupled systems – thermal reactors – in the late fifties clearly showed the limits of the PK model and the need for more accurate solutions with regard to reactivity, time-dependence of power distribution, etc… induced by some locally triggered external event. Several modifications were brought to the early PK model, to cope with its inefficiencies in describing reactivity transients in loosely coupled systems. This lecture will recall the basic properties of the PK model and will review its methodological refinements leading to a set of different numerical schemes collectively known under the name *quasi-static* (QS) approximation.

Overview of the quasi-static approximation

Allan Henry is widely regarded as the father of the QS method despite the fact that he didn't coin its name. The basic idea appeared in two papers he published in 1958 (see [2,3]). He attempted to unify the approach of tightly and loosely coupled systems using a factorization of the neutron flux, solution to the transport equation, into the product

$$\phi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t) = T(t)\,\psi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t) \qquad (1)$$

of two functions called `*amplitude*' and `*shape*' respectively. The former depends only on time and provides the bulk information about the power change, whereas the latter describes - with proper normalization - the time-dependent deformation of the power profile. Using this splitting, Henry related the amplitude $T(t)$ to the point kinetics equations and gave explicit relationships for the PK parameters in terms of $\psi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t)$, the shape function. However, to the important question about the relation between $T(t)$ and $\psi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t)$ – given that their product satisfies the transport equation – he gave only a rather crude answer. The shape was to obey a transport equation - not the correct one - with perturbed cross sections. In the author's mind this would hopefully give PK parameters in better agreement with observations of actual systems behaviour. Numerical experiments reported in a later paper showed that this so-called `*adiabatic*' approach of reactor kinetics gave much better results than PK on a loosely coupled system typical of light-water moderated power reactors with a two neutron-groups diffusion description [4]. However, the discrepancy with a full space-time calculation (obtained using WIGLE) was still considerable, even greater than with other approaches like modal synthesis [5].

A few years later, Ott and Madell showed that the discrepancy between the adiabatic approximation and exact results could be significantly reduced by a treatment of the nonlinear coupling between amplitude and shape [6]. They called the resulting scheme `*quasi-static*', the name used today.

Further improvements with regard to the treatment of the time derivative of the shape function came a little later in a paper by Ott and Meneley [7]. This gave the quasi-static method its present form, at least with regard to the implementation in many industrial computer codes, known as the `*Improved Quasi-static Method*' (IQM). Numerical experiments on fast and thermal reactor cores showed convincingly the improvements over previous methods and many studies were undertaken to assess the accuracy of the results, adding further thermal-hydraulic feedback [8].

IQM is essentially a nonlinear approach of the time-dependent basic transport or diffusion equations. Its development lacks mathematical rigor. For instance, it doesn't tackle the equations using the Newton-Raphson method, a common practice for nonlinear equations, but in a more or less *ad hoc* way. This triggered further research (see [9, 10]) that eventually led to the development of the *generalized* quasi-static method. Here the nonlinearly coupled PK and shape equations were dealt with a Newton-SOR method [10, 11]. The method gave good results for fast transients, showing convergence with increasing number of time-steps. The implementation of a full version being very sophisticated, the use of the method did not go beyond numerical experiments.

Early studies of the quasi-static approximation and application to power reactors were made in the multigroup diffusion framework. Computer codes such as QX1 and FX2 were built specifically for such purposes. Later on, IQM modules were introduced in large diffusion 2D/3D solvers such as DIF3D-K, Kiko3D or MINOS. The coupling of the IQM approximation with transport codes went also through, at a slower pace though. The well-known transport discrete ordinates code DORT has a time-dependent companion called TDDORT with an IQM module [12]. A quasi-static module can be found in the cell code WIMS-AECL RFSP, and has been implemented in the Canadian transport code DRAGON [13]. These examples are given for illustrative purpose but the list is far from exhaustive.

Since the start, the rationale of the QS method has been to spare computer time, relying more on the information provided by PK than on the full basic - transport or diffusion - equations. However, since the PK parameters closely depend on the flux distribution, the accuracy of the latter is crucial. In IQM one evaluates the PK parameters using the shape function computed on a *macro time-step* Δt and one solves the PK equations on a *micro time-step* δt. Hence, in addition to the normalization constraint, one uses an elaborate iterative scheme to solve the nonlinear coupled PK and shape equations. Despite the intricacies, there is a hope to achieve computer savings.

IQM is based on the *amplitude-shape* $\{T(t), \psi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t)\}$ couple. One might instead consider the *amplitude-flux* $\{T(t), \phi(\boldsymbol{r}, E, \boldsymbol{\Omega}, t)\}$ couple and use ψ as an auxiliary tool to fulfill the normalization condition required by (1). Although known for some time this dual approach of the quasi-static approximation has received little attention up to now with regard to computation efficiency, and is being currently revisited [14].

Finally, GEN-IV innovative systems have brought a new impetus to quasi-static studies both in methodology and applications. So far the time-dependent transport equation to be solved assumed neither the content of external neutron sources, nor nuclear fuel motion. Interest for accelerator driven systems (ADS) with high energy spallation neutrons and the revival of molten salt reactor concepts have introduced new challenges to the method as it is used today. Recent numerical experiments show that situations exist with such nuclear systems where the convergence of the iterative scheme is so slow that the computer time for some transients is larger than the one entailed by a direct technique using the most suitable implicit algorithm [15].

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