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TOPIC ③

NEUTRONICS & CORE PHYSICS

3.1 Introduction, current code capabilities

Introduction to transport theory for core physics calculations

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Introduction

The neutron transport equation constitutes the fundamental model for the neutronic evaluation and the design of nuclear reactor cores. The transport equation for neutrons is a linear version of the original kinetic equation for gases written by Ludwig E. Boltzmann in the late nineteenth century. The equation is of integro-differential type and for realistic cases of interest for nuclear engineering it is solved by numerical procedures implemented in high-performance computational codes. This lecture is devoted to review the current status of the computational methods and their implementation into computer codes for the solution of the neutron transport equation for reactor physics applications.

In the past the direct solution of engineering problems by full transport methods was restricted by the limited computing capability. Therefore it was necessary preliminarily to highly simplify the transport model, down to approximations that might significantly distort the physics of the problem and produce unsatisfactory results. The diffusion model was heavily utilized, especially for multidimensional evaluations. Time dependent calculations were carried out by very crude models, down to point kinetics. The tremendous computing capacity available today is often making possible to solve realistic problems to a high degree of accuracy with a minimum of model approximation, accounting for the full multidimensional geometrical and material complication.

The transport problem can be approached through different mathematical formulations, which, although physically equivalent, may yield specific numerical advantages. Aside the original integro-differential equation characterized by a first-order space differential term, also the second-order and the integral forms can be used.

Overview of methods for neutron transport and code capabilities

The transport equation is reduced to systems of partial differential equations by a preliminary discretization of the energy variable, leading to multigroup formulations of the model, and by either spherical harmonics expansions or direct discretization for the angular variable, leading to P_N or S_N methods, respectively (Lewis and Miller, 1993). Several variations to these basic methods have been developed over many years. The spatial model is then tackled by various discrete schemes. The starting point may be either the first order or the second order equations. The choice of the second-order equation may be motivated by the possibility to apply ad-hoc variational properties useful for the development of numerical techniques. It has also been shown how the simplified spherical harmonics method can be related to a second-order (A_N) formulation (Ciolini et al., 2002) of the transport equation. From the second-order equation response-matrix formulations can be readily obtained for efficient applications to full-core evaluations.

More recently, the method of characteristics has become an efficient and powerful tool for transport analysis and its potential can open new and interesting possibilities for multidimensional problems and for unstructured geometries. Neutrons are tracked along their line of motion with a large geometrical flexibility. Enhanced numerical techniques have been devised for particle tracking (Le Tellier and Hébert, 2006; Le Tellier and Hébert, 2008). Also the acceleration of numerical schemes is an important issue that has been taken into consideration (Grassi, 2007; Santandrea, 2007). Characteristics methods are quickly becoming standard tools for reactor physics calculations and are available in various code systems, such as APOLLO and DRAGON.

The latest years have certainly seen significant and numerous developments of new numerical schemes. It is worth referring briefly to some new developments on classical schemes. New

schemes could soon lead to pin by pin evaluation capabilities without homogenization (Masiello and Sanchez, 2007). Efficient solvers for the spherical harmonics model have been also implemented in industrial codes (Baudron and Lautard, 2007).

Some new developments have also been made in the variational nodal method (Zhang and Lewis, 2006) exploited in the code VARIANT. At last, interesting and promising new approaches for the angular discretization using wavelets have been proposed (Buchan et al., 2008), which would constitute a real novelty in the field.

The presentation will contain a short review of the applications of various methods in numerical codes, highlighting the specific field of application. In particular, the features of the codes used for the successive steps of reactor design will be illustrated, starting from spectrum calculation for the generation of average few energy-group data to full multidimensional core evaluations.

Benchmark activities are of foremost importance in the field of computational transport. They may provide a rich insight into the physics of the transport process, and a validation tool to qualify models and numerical methods as well as to ascertain the limits of applicability and the shortcomings of a computational procedure.

The assessment of the performance of computational tools by comparisons of the results independently obtained for a significant problem is a well-accepted and recognised practice. Recently the outcome of the benchmark proposed by E.E. Lewis has been published. This benchmark is intended to test the capability of the tools used by the various participants to carry out a full core calculations without performing any spatial homogenization on fuel and coolant. Both two dimensional (*Progress in Nuclear Energy*, 2004) and three dimensional configurations (*Progress in Nuclear Energy*, 2006) are analysed. The reading of the papers included in these special issues proves extremely interesting and useful and is very instructive to highlight specific
properties of the various techniques. Also for fast assemblies a transport benchmark without homogenization has been proposed (Chiba and Numata, 2007).

Cross section generation

Cross section generation is a fundamental step in reactor analysis in order to provide meaningful parameters for transport evaluations. Over the years the standard procedure, contemplating successive steps in the spatial homogenization and energy averaging, passing through the different levels of heterogeneity, has been established and tested. Such procedure can be viewed as a spatial multi-scale approach. Homogenized average cross sections are usually evaluated on the basis of reaction-rate conservation principles. However, different principles have been introduced in special applications (Ragusa et al., 2007).

The methodology is well-assessed over so many years during which it has been applied to commercial systems. However, some challenges may come forward for the analysis of innovative systems. For instance, in the analysis of problems involving very high energy neutrons, suitable techniques should be devised to avoid unphysical shortcomings associated to Legendre polynomial representations of the angular scattering functions that may lead to the appearance of negative terms (Kim and Cho, 2007).

Another interesting problem in data generation is encountered in the production of few group cross sections for pebble bed high-temperature reactors (Ilas et al., 2006), for which complications come into play owing to the double heterogeneity of the fuel elements.

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